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SCALE RELATIONS AMONG SAND-BED RIVERS INCLUDING MODELS

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HYDRAULICS DIVISION

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SCALE RELATIONS AMONG SAND-BED RIVERS INCLUDING MODELS

Thomas Blench,[1] M. ASCE

SYNOPSIS

After a Section on models in general the value and practicability of model simulation of the water-sediment complex of sand-bed rivers are discussed. A Section explaining the origin and application of regime theory formulae to rivers is then presented as an introduction to an example of their use in designing the initial scales of a model that possesses a bed-load of natural river type. The reliability of the initial scales, and the impossibility of avoiding some secondary adjustment are discussed, and followed by examples of calculations aimed at reducing the number of secondary adjustments to a minimum; a Table of useful derived scale relations is proferred for this purpose. The Conclusion recommends the controlling of important models by a panel representing a variety of skills.

INTRODUCTION

Although the utility of mobile-bed river models is accepted generally, complaints may be heard that:

a) They may not yield results till years have elapsed.
b) The results may be trivial.
c) The cost may be high.
d) The quantitative predictions may be uncertain.
e) The differentiation of bed-sand into offtakes may be poorly represented.
f) The rate of sediment movement, and the associated rate of development of bed configurations, may be grossly misrepresented.
g) An experienced river engineer could predict some aspects of river behavior better than a model.

The author believes that the situation disclosed by these complaints could be improved if model-makers used the relatively new weapon of regime theory (1-3) to design and adjust model scales. The same weapon permits the engineer to analyse river data and use the results to answer many questions without models; in fact, he can use one river as the model of another. Therefore, the second half of this paper describes and uses regime formulas instructionally, and the whole is written as much for the practising engineer as for the model-maker.

Models in General

The intention of a model is expressed well by the non-technical description "a small one that does just the same as a big one." As the engineer prefers

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quantitative predictions from models, and knows that slavish copies are not always necessary, the following formal definition is proposed for present purposes:

> "A model simulates a set of physical occurrences, called the prototype, with the object that the measures of certain physical entities in the prototype should be known constant multiples of the same entities at corresponding points in the model."

The constant ratios are called "scales" and the model quantities are usually in the numerator. For example, if the long-term average discharge at a place in the prototype is a million times that in the model then the discharge scale is one millionth, and applies to all discharges at all times and places—assuming, of course, that the model has achieved its object.

As models are built because of ignorance their design and operation call for a set of skills seldom found in one individual—inventiveness, analytic insight, outstanding knowledge of dynamics, wide experience of prototypes, mechanical ingenuity, optimism, and a cooperative temperament, to mention some principal ones. Unless these skills are available to a difficult model there can be little hope of success.

When the model-maker—to put matters rather paradoxically—is not completely ignorant of his ignorances, he can devise his model to eliminate factors of ignorance and improve the possibilities of scaling other factors. An example will be given from rigid-boundary work which is relatively simple and well developed. Suppose a drop structure is to be modelled. Ignorance of the effect of shape is eliminated, to a first approximation, by making the shape the same in model and prototype—in other words, length scales in the three coordinate directions are made equal. To a second approximation the shape should apply to every irregularity of the boundary, so that boundary roughnesses should also be to length scale; as this is normally not practical, and might not be very useful unless some other matters receive attention, the modeller usually neglects it and hopes it does not matter. Ignorance of the relative effect of gravity and inertia is eliminated by making the Froude Number the same in model and prototype, that is by running the flow at a calculated speed; for dimensional considerations, having nothing to do with hydraulics, show that the scale of the ratio of gravity to inertia is unity when the Froude Number scale is unity. Ignorance of the relative effect of fluid friction and inertia is shown, by the same non-hydraulic considerations, to be removeable by running the model so that the Reynold's Number is the same as in the prototype. At this stage an insuperable difficulty arises, for, if the Froude criterion requires the model flow to be slower than the prototype's, then the Reynold's criterion requires it to be faster. So the model-maker hopes that the Reynold's criterion does not, in fact, matter very much and ignores it. He then uses the same non-hydraulic considerations to show that, if he had followed both criteria, the streamline pattern in the model would be geometrically similar to that in the prototype; so he hopes it will be and proceeds to run the model to the Froude criterion alone. Naturally his duty is to compare model and prototype results as soon as possible. Experience shows that reproduction is quite good for some important occurrences, e.g. discharge coefficients, and position of hydraulic jump when not complicated too badly by air entrainment; it can be quite bad if divergence of flow after the jump is under study, for divergence of flow depends principally on boundary resistance and fluid viscosity, and the model maker has failed to do anything about making the factors related to them equal in model and prototype.

From this example it should be clear that the efficiency of models can be increased greatly by additions to quantitative knowledge. For example, if the laws of friction for divergent flow were known, even approximately, the model-maker could form an estimate of how the prototype flow might deviate from that of the model, or might even be able to alter the model's scales so that the flow representation would be correct. The reason why a few major mistakes have punctuated the progress of rigid-boundary modelling to its present respectable status is also obvious.

The modeller of sand-bed rivers starts with the major handicap that, because the boundary material is all-important, he cannot get rid of the shape factor by keeping overall shape the same and ignoring the boundary material; he would have to scale the boundary material according to length scale, and this would normally result in using clay-size, and therefore cohesive, material to represent non-cohesive sand. In any case there is little comparison with rigid-boundary conditions. The fluid is no longer simple; it is a water-sediment complex, in which the sediment is generally in two principal and quite different phases, viz, suspended load and bed load. The major portion of the solid boundary, so-called to distinguish it from the free surface, is really the bed-load; the bed configuration is one of moving dunes, and the particles in it move by rolling and jumping. The particles of the bed are of various sizes, that are found to be distributed according to a definite average law closely resembling the error function in terms of the logarithm of particle size. The minor portion of the boundary, forming the sides, has the flow along it in a phase quite different from that along the bed. The suspended load, in nature, is of unpredictable constitution, and can have a marked effect on boundary roughness, even in laboratory flumes with rigid boundaries. The two principal sediment phases and the water phase interact on each other. As might be expected these interactions are reflected in a remarkably simple set of relationships among the various self-adjusting average dimensions of self-forming channels that have achieved equilibrium, and these relations allow overall parameters to be defined so as to link with the general nature of the water-sediment complex—just like rigid-boundary roughness is defined from the observed functional relationship between velocity, hydraulic radius and energy gradient. Work is in hand relating the major parameter to various properties of the water-sediment complex—again just as rigid-boundary roughness could be related to roughness heights, shapes, spacing and size distribution if the labor were thought justifiable.

Although quantitative information on the preceding topics is vital to successful modelling and is available [1-7], it has not been all coordinated into a single text, some is written in difficult terminology, and it is all rather too close to be seen in perspective. Therefore the following portion of this Paper suggests, for consideration, a sequence of points that the new knowledge has shown to be relevant to the removal of factors of ignorance and (after these factors have been removed) to the calculation of model scales. For reasons implicit in what has been said already the author has not the temerity to offer suggestions on the many facets of model construction, nor the hope that any practical model can reconcile more than a few of the theoretical requirements for constant scale ratios for all entities. He does hope that no model-maker will ignore the points raised.

Grain-size distribution. The bed-sand of movable-bed rivers in equilibrium has adjusted its grain-size distribution to conform to a fairly definite law, so that, if mechanical analyses are plotted on logarithmic probability paper, the results for any one river will be a series of approximately parallel straight lines.[6, 7]

If a model does not use bed-material of natural grain-size distribution the following defects of behavior may be expected:

a) The sand constitution will change till it is natural, unless it is of grains all of the same size.
b) Differentiation of bed-load into offtakes will be unnatural.

A practical source of natural sand is from the river that is to be modelled. Analyses of it should be recorded on logarithmic probability paper for control purposes. Pit sand may be found to conform to the distribution law found from a river. Sand from dredging piles is often a mixture and unsuitable.

Sand size for non-tidal models. Sand size is best taken as the median from a mechanical analysis in terms of "percentage by weight coarser than," i.e. the "50% size." For many purposes sand-size is of little or no relevance; in fact, an expert river engineer with knowledge of regime theory may use, as his model of a particular phenomenon expected in a given river, a similar occurrence in another river. However, if a formal model is to be constructed, the following consequences of using sand size different from the prototype's should be considered:

a) Because of present ignorance of the exact relation between bed-factor and sand size, exact calculation of model scales will be impossible.
b) Sediment discharge scale will differ by an unknown amount from water discharge scale.
c) The differentiation of bed-material into offtakes may be unrepresentative.
d) If sand-size in the prototype varies with distance, or is different in different channels, there is no rule for scaling these variations in a model that does not copy prototype sizes exactly.

Sand size for tidal models. In a tidal model the consequences of not using the prototype sand are as in a non-tidal one. However, to ensure that tidal currents scale correctly, the Froude number in terms of depth must be the same in model and prototype. This Froude number, multiplied by g, is the bed-factor of regime theory and, as such, is known to be a function of bed-sediment size, relative density and charge. Therefore, and because the charge in a river likely to merit modelling will usually be small, a river sand that is appreciably coarser than the prototype's is unlikely to be suitable in a model. On the other hand a finer river sand can always meet the requirement, and so may a coarser light-weight artificial "sand" such as coal or pumice.

Fluid viscosity. Although viscosity is usually assumed to have but slight effect on regime relations it is a factor in defining the water-sediment complex and varies considerably, in the field, during a year. Pending results from research there is always a possibility that a big temperature difference between model and prototype water ought to be compensated by a slight difference in sand size or a variation in scale.

Considerations in Bed-sand Injection into Models

To ensure that rates of bed-sediment movement, and therefore of degradation and aggradation, are correctly modelled, an essential condition seems to be that the sediment discharge scale should be the same as the water discharge scale; that is, the charge (viz. ratio of weight per second of sediment to weight per second of fluid) should be the same in model and prototype. Now, if the model is to have definite fixed scales for the different variables, the Froude number scale (i.e. the bed-factor scale of regime theory) must be fixed. But the bed-factor is a function of bed-material size and of charge, so the bed-material must have a size fixed at a certain fraction of the prototype's. Apparently, therefore, so far as regime behavior is concerned, and provided that the bed-factor scale does not have to be unity to suit appreciable tidal currents, there is no reason why any natural sand within the logarithmic probability gradation range [6, 7] should not be used to model any other in the same range.

However, although agreement of model and prototype under regime conditions is necessary, it is not sufficient when rates of development are in question. A main reason for insufficiency arises if high flood can produce local velocities in excess of critical in the prototype. (At critical velocity Froude number = 1.0, or bed-factor = g.) At critical velocity dune-movement changes to sheet movement of the bed and the law of transport suffers a discontinuity. Another main reason is that deviations from regime that put material temporarily into suspension (or even undeviated conditions, such as at an offtake head, that do the same) bring settlement velocity into the picture. Reasons such as these require the additional condition that the model bed-material be the same as the prototype's. If it is not met reproduction of rates of bed-movement cannot be expected, nor can exact reproduction of such phenomena as differentiation of sediment into offtakes.

The practical question of how much sand to inject to represent prototype bed-load conditions, with the restrictions now imposed, still remains very difficult. There is no satisfactory way of measuring a river's bed-load. Even if there were, load variations would probably be so erratic that several years of daily observations would be needed to give a fair picture. Fortunately, violent bed-load variations, in practice, do not seem to produce violent reactions from a river unless they are greatly prolonged; so the model-maker need not contemplate exact reproduction, and can start with a rough estimate of the overall picture. Dredging figures can give one estimate. Total load observations might exist at a turbulent site upstream, and give another. Yet another can come from application of the well-known Meyer-Peter formula and Einstein curve. Regime analysis of channel dimensions gives another, and will be much more powerful when—as is expected soon—sediment transport formulas in terms of its parameters are published. With estimates of average, or smoothed, charge derived as indicated the model-maker should be in a fair position to start using his ingenuity.

The following tentative procedure is suggested for consideration, to ensure that the points dealt with above receive attention in a model that aims at reproducing rates and differentiation of bed-load movement to scale under general conditions:-

a) Use prototype bed-sand in the model.
b) Design the model scales with Froude number in terms of depth the same in model and prototype.

c) Estimate, by all available means, the likely average bed-load of the prototype and, thence, the average charge of bed-load.
d) Use prototype bed-sand for initial injection if there is little variation of median size with distance in the reach to be modelled; bed-sand size should be estimated from main channel bed. If there is important variation of median size with distance start with sand estimated from the finest.
e) Unless experience has shown how to arrive at a better start, start tests with charge at all discharges equal to assessed mean prototype charge. Tests should be run with the object of reproducing a major short-term bed activity, such as scour and bed-wave development as observed by sonic sounder during the course of high flood.
f) Modify injection rates to vary with discharge and, as reproduction of the known circumstances improves, consider modifying sand grade also.

A subsidiary suggestion for consideration is that the model should be designed to give the same, or comparable, tractive force intensity on sides as occurs in the prototype. The reasons for this are still not satisfactory and will appeal only to an expert; they are discussed in the rather mathematical section at the end of this paper. There is a major practical difficulty that the imposition of the condition leaves only one more in the designer's hands and, for practical reasons, he is likely to want two.

It is emphasised that the preceding considerations are of major dynamical principles that the model-maker cannot afford to ignore, in the general case, except at the risk of major distortions of his reproduction of rates of bed-movement and of sediment differentiation. It is to be expected that a variety of other considerations, outside of the scope of this paper, will compel the acceptance of some distortions to minimise others. Unless a model is of specialised and limited type it will have more features than can be reconciled completely by any system of procedure.

Considerations in Scaling Suspended Load

In canal practice a suspended load of some 1% by weight of the water may upset rating curves by about 5%, and will affect regime analyses as if there had been some 20% change in fluid viscosity. Mechanical analyses of these loads are not on record, but they may grade all the way from clay up to bed-sediment size on one sample. Vito A. Vanoni records[12] the effect of suspended load on friction coefficient in laboratory flumes, and offers an explanation. There does not seem to be any reliable field information on whether suspended load in practical quantities affects bed-transport seriously. Judged by model work done in India with clean water representing rivers having sediment loads very much like those of the Colorado in Grand Canyon, suspended sediment does not seem to have much effect; on the other hand river models all over the world have avoided useful quantitative predictions of sediment movement so completely that important effects could escape notice.

There is no doubt about the importance of suspended load in the distribution of bed-load into offtakes, and in the formation of deltas and shoals. When there is only bed-load, the exclusion of practically 100% of bed load from offtakes is obtained easily by suitable design; when the suspended load contains sizes comparable with those found in the bed load, sediment exclusion from offtakes is highly inefficient, since suspended load cannot be excluded and provides bed-load by differentiation out of itself. Some practical

problems are concerned almost entirely with shoal formation in estuaries,[11] and suspended load is then all-important.

On all scores the introduction into a model of suspended load comparable in intensity and constitution with that of the prototype is desirable whenever it is likely to be relevant. There are no data to indicate the least amount of load likely to have an effect; the author would not expect 0.1% by weight to have appreciable effect except in delta formation, but would regard 1% as important. Fortunately methods of sampling suspended load in the field are well developed, and the simulation of an average suspended load based on field information should give no trouble to a model-maker.

Viscosity of the water may have practical relevance, as discussed under "Considerations in Choosing Sand for Model Beds."

Fundamentals of Regime Theory

As the equations of regime theory[1-3] afford a means of removing several factors of ignorance from river models, and thereby permit the scaling of factors of major engineering importance, a brief resume will be given here.

The theory concerns all channels that carry a sediment load and are capable of adjusting their solid boundaries. It accepts, as a principle, that if the fluid discharge and the constitution of the water-sediment complex of a given channel fluctuate about long-term means, then the self-adjusting variables defining the channel behavior will fluctuate about long-term means that are determined by (a) the mean discharge (b) the mean nature of the water-sediment complex (c) the nature of parts of the boundary that may be physically unrelated to the water-sediment complex's constitution. The self-adjusting variables regarded as adequate for engineering description of a river are breadth, depth, slope, meander length and meander breadth; for an artificial canal there will be no meandering, so only the first three variables remain. Any channel as described is said to be of "regime type"; when its behavior can be identified as fluctuation about a mean state it is said to be "in regime."

The quantitative object of the theory is to find dynamically satisfying equations, consistent with observed behavior, that relate the self-adjusting variables to the independent ones (a) to (c), or to each other as an intermediate step.

It is most important to notice that, for each self-adjusting variable, there must exist a physical equation; and if the variables can adjust independently of each other, the equations must be independent of each other. The terminology of dynamics expresses this fact vividly by calling the number of self-adjustable variables the number of "degrees of freedom" of a channel. A clean water flow in a rigid channel has one degree of freedom only, viz, with respect to depth.

The data of regime canals with fairly steady discharge, no meandering, cohesive sides, and bed-material not coarser than sand, have yielded three equations that are susceptible of extension to river problems, with precautions. Some of their major showings are (a) the Froude Number in terms of depth is the same for all channels of the same water-sediment complex, (b) the sides behave as if "smooth," (c) the friction factor defined as in rigid-boundary hydraulics but with depth used instead of diameter, varies inversely as the fourth root of the Reynold's Number in terms of breadth—so that the "smooth" (meaning "formed from the fluid") boundary of rigid boundary hydraulics is just a special case of a boundary formed from a water-sediment

complex, the form of the Blasius Equation applying to both. As such channels retain a form that is very closely trapezoidal, in spite of bed-dunes, the formulae are expressed conveniently in terms of the depth, d, from water-surface to bed, and the breadth b that multiplied by d gives the cross-sectional area of flow. The first is:

$$V^2/d = F_b \tag{1}$$

which uses a Froude Number in terms of depth to define a bed-factor F_b; V is mean velocity of flow. The second defines practical side-factor by:

$$V^3/b = F_s \tag{2}$$

where F_s is this side-factor. Actually V^3/b should be multiplied by the square of the mass-density, and by the kinematic viscosity, to obtain a measure of the square of the tractive force intensity on the sides. The third equation is

$$V^2/(gdS) = C(Vb/k)^{0.25} \tag{3}$$

where S is the surface slope, C is a non-dimensional constant approximately equal to 3.63, and k is the kinematic viscosity of the complex. Investigations outside the canal range have shown C to be a function of charge.

Practical purposes are best served by manipulating the preceding equations to give three alternative independent ones, each in terms of independent variables Q, F_b and F_s, where Q is discharge in cusecs (i.e. cubic-ft. per second). The results are:

$$b^2 = F_b Q/F_s \tag{4}$$

$$d^3 = F_s Q/F_b^2 \tag{5}$$

$$S = \frac{F_b^{5/6} F_s^{1/12} k^{1/4}}{Cg \quad Q^{1/6}} \tag{6}$$

These equations permit the canal designer to decide the regime dimensions of any channels for which he knows the discharge and the nature of the water-sediment complex and sides (which may be part of the complex), as defined by F_b, F_s and k. Their application to rivers requires careful qualifications, for which the reader must study the original theory and references; roughly, it may be said that they hold provided Q is a suitable mean, and with coefficients introduced to correct for the particular mean, the effect of meandering, and heterogeneity of bank conditions. Fig. 9 of Ref. 4 provides most

striking confirmation of how the rulings (a) breadth varies as root Q and (b) depth varies as cube-root Q are confirmed by gauging sites on American regime-type rivers, provided Q is the long-term arithmetic mean discharge; the deducible rule that V varies as sixth root Q is naturally confirmed equally well by the same figure.

For meandering, analysis of rivers by Sir Claude Inglis[10] shows that both meander length and meander breadth vary as square root of discharge; he elected to use peak flood discharge as a parameter. No doubt the coefficients are closely related to that of equation (4) modified by a factor to allow for hydrograph form, and nature of banks. Obviously if this law did not hold the universal practice of making the plan view of a model a photographic reproduction of the plan view of the prototype would have led to difficulties long ago. This practice resulted from convenience, but there was no reason to expect, ab initio, that it was correct; when models were made with the vertical scale the same as the breadth scale—also, no doubt, for convenience or for lack of any guiding principle—they refused to reproduce prototype behavior.

Initial Model Scales Calculated from Regime Equations: An Example

If the water-sediment complex of a model is of natural river-bed type, as explained in previous Sections, then the model is truly a small river. Therefore, equations (4) - (6), suitably qualified, must apply to it just as to any other river; the two meander relations will apply also. Even if the equations do not represent river behavior perfectly, the fact remains that exact equations of very similar type would; models that are built without consideration of the existence of the equations cannot be expected to run as intended but, instead, will tend to adjust themselves to conform to the laws expressed by the equations.

The application of the equations to scaling, both practically and mathematically, is essentially that used in rigid boundary work where the Froude, Reynolds and other criteria replace definite laws. For example, suppose the model-maker had 2 cusecs available to model a river with floods up to 100,000 cusecs, so decided on a discharge scale of 1/50,000. (Note that "scale" will always be taken as ratio of model quantity to corresponding prototype quantity.) To decide on a breadth scale he would take the regime equation containing breadth, viz. eqn. (4) and write it as:

$$b^2 = C_b F_b Q/F_s \qquad \text{(4a. scale)}$$

C_b is a coefficient introduced to allow for hydrograph form, peculiarities of side conditions, meander pattern, and nature of suspended load as major factors that might alter conditions from those to which the canal formula applied. (The word "scale" after the equation number means that every symbol in the equation should be taken as a scale, and not as a measure, so that, in the present example, Q means 1/50,000. This convention avoids the nuisance of writing b_m/b_p, Q_m/Q_p etc. etc. into every equation derived by writing a regime one with suffix m denoting model, rewriting it with suffix p denoting prototype, and then dividing one by the other. Obviously, with this convention, a scale equation is just a basic equation with the true constants left out—e.g. equation (6) would have C and g omitted, since they cancel on dividing to get scales.) As Q is the only term known in equation (4a. scale) the designer has

to decide on some more before finding a suitable b scale. If the model were tidal he would be compelled to run it so that $F_b = 1.0$, since the F_b scale is just the Froude-Number-in-terms-of-depth scale: otherwise there would be a limited range of choice. As there are good reasons for making $F_b = 1.0$ in any case, e.g. those given in preceding Sections on water-sediment complex, let us assume he makes it so. Attention then comes to C_b and, as it is a very vague quantity, the best thing to do is to treat it as a coefficient of ignorance and try to arrange matters so that it shall be 1.0. Obviously one step in the right direction is to make the meander pattern similar in model and prototype; but this is just the conventional thing to do anyway. Another step, if the designer believes that channel breadth in nature scales exactly like meander dimensions, is to make $F_b/F_s = 1.0$, which means $F_s = 1.0$ in this case. So, in the end, he decides to make the F scales each unity in the hope that this will satisfy some special requirements and will also ensure that C_b is very nearly unity. The b scale, to suit, has to be the square root of the Q scale, viz. square root of 1/50,000 = 1/224. Then equations (5) and (6) give the depth scale as the cube root of 1/50,000, viz. 1/36.9 and the slope scale as the sixth root of 50,000 viz. 6.08. This completes the information needed for building the model.

At this stage the reader should notice that the designer disposed of the two meander equations by making the scales for meander length, meander breadth, and channel breadth all the same. This left him with three equations (4) - (6) at his disposal, so that he was at liberty to fix three and only three scales; he chose the Q, F_b and F_s ones for fixing, but could have taken any other three. After fixing the three all he can do is find what the remaining scales will adjust themselves to, if he can physically impose the scales he fixed mathematically. One object of finding the self-adjusted scales is, of course, to anticipate nature and thereby avoid months of effort wasted in trying to make the model run in an impossible manner.

For further scaling the following equations, derived from equations (4) and (5), are useful:

$$V^6 = F_b F_s Q \qquad \text{(7. scale)}$$

$$(d/b)^6 = F_s^5/(Q.F_b^7) \qquad \text{(8. scale)}$$

$$T = W/V = (F_b.Q/F_s^2)^{1/3} \qquad \text{(9. scale)}$$

Strictly, each equation should have a "coefficient of ignorance," but it is usually omitted because the model-maker arranges matters, as previously described, so that it should not differ much from unity. T, defined by W/V, is merely a representative time, chosen to be calculable from the equations so as to give a time-scale. If rates of change of discharge are regarded as important, the time scale tells how to impose the model river's hydrograph; for tidal models it gives the model tidal period as a fraction of the prototype's. So far as the equations are concerned the time scale, or the velocity scale (which is effectively the same thing), applies to the water flow. However, if bed-factor is kept the same in model and prototype, and the model's bed-material is the same as the prototype's, then the charge must be the same

for both, so that sediment discharge scales just like water-discharge; presumably the time scale will then be the same for bed-load as for water.

The d/b scale is also called the "vertical exaggeration." The smaller the Q scale the bigger the vertical exaggeration and the less representative are conditions near the sides and round rigid obstacles.

Scales found from equations (7) to (9), together with those already calculated, are summarised below:

Imposed Scales. $Q = 1/50{,}000$; $F_b = 1.0$; $F_s = 1.0$

Consequent Scales. $T = d =$ cube root $(1/50{,}000) = 1/36.9$

$d/b = S =$ sixth root $50{,}000 = 6.08$

$V =$ sixth root $(1/50{,}000) = 1/6.08$

Some Practical Considerations in Imposing and Calculating Scales

Two major practical questions arise from the previous section:

a) How can a model-maker "impose" scales?
b) What reliance can be placed on the scale calculations?

The answers will be illustrated from the preceding example. Obviously the discharge scale can be imposed with great accuracy if prototype hydrograph and tidal data are available and the modeller is prepared to copy them. The side-factor scale cannot be imposed directly, for it is a scale of tractive force intensity on sides; however, by giving the model the deduced breadth scale, the side-factor scale will look after itself provided the model runs otherwise according to plan. The bed-factor scale can be imposed only by using a variety of water-sediment complexes, and checking how the model reproduces known prototype behaviors; when it reproduces them correctly the imposition may be regarded as achieved. Obviously the use of the same bed-material as in the prototype will simplify this process, for (since suspended load is of secondary importance, so far as is known at present) the model-maker will need to adjust only the injected charge, and he ought to have a fair idea of what it should be. It is difficult to see how a model-maker can avoid some indirectness in imposing scales, for, in any case, he does not know the bed-factor of the prototype. Suppose, for example, that he is uninterested in rates of sediment-movement and the model is not tidal. Then he might like to use an arbitrary river-bed sand combined with an arbitrary injection rate. If the skills desirable on a model were available (see Fundamentals of Regime Theory) he would have an estimate of the bed-factor he was going to impose, and could arrange for flume experiments to give it with quite good accuracy; he would also have an estimate, not so good, of the bed-factor of the prototype. There would remain two, and only two, imposable scales. Suppose he chose width and depth, and they led to reasonable values for the other scales; then, even though the theory were perfect in application, the model as built and run could not perform correctly unless the estimate of prototype bed-factor, from which the F_b scale was calculated, happened to have been correct. Probably the simplest practical course would be to run the model with the width, discharge and time scales as planned, let it adjust by degradation or aggradation, and then accept the self-adjusted depth and

slope scales. Whatever pair of imposed scales were substituted for breadth and depth the same difficulty would arise.

So the general answer to the first question is that, even were the scaling equations perfect, some scales have to be imposed indirectly under circumstances that will involve arranging for the model to be allowed to adjust; if the requirements of the model are stringent the adjusting process will involve manipulation of one variable till agreement is obtained between a test occurrence of the prototype and its representation by the model.

The answer to the second question is that, provided a model is devised to be truly a small river (using an unnatural type of sand would prevent it from being one), the reliability is indicated by the applicability of the regime equations to rivers. A numerical measure of accuracy cannot be given, but the author would say that it is very good practically. Published samples of applicability are in References (4), (9) and (10). Good applicability seems to be shown by the fact that the rule of "depth scale = two-thirds power of breadth scale" found from experience[11] is a deduction, for $F_b = 1 = F_s$ from regime theory (see previous section). A plot, on double logarithmic paper, of the depth against width scales of a variety of well-known European and U. S. waterways Experiment Station models[8] shows how well cut-and-try methods have forced scales to fit the more exact regime theory scale requirement:-

$$d^3 = (F_s^2/F_b^3).b^2$$

with $F_b = 1.0$ and F_s approximately = 1.0 for tidal models. Theory seems to indicate[8] that, for a tidal model, the F_s scale should be the square root of the viscosity scale, but has gone somewhat ahead of well-established arguments so cannot be discussed here. As with any other calculation, the operator must understand the theory if he is to avoid misapplication. The same precaution is needed in rigid-boundary hydraulics where an operator who does not understand principles will use a rough-boundary formula for a smooth boundary case, apply a backwater function to find the shape of flow over a sharp-crested weir, and so on; and there is only one formula to misapply to rigid boundary hydraulics. The particular reason for the warning is that, no matter what precautions are taken to make the model a real river, there will be elements that do not reproduce prototype conditions, even though there is nothing unnatural about them in the sense that an arbitrarily graded bed-sand would be unnatural. Thus, a major source of trouble is that small scale means big vertical exaggeration; in the worked example the vertical exaggeration of about 6 corresponds to a horizontal scale of about 1/225. The "sides" of a river section might be defined by the property of being free from bed-dunes and, particularly in a tidal river, might be of sand at comparatively flat slopes, say 1 upon 2. A vertical exaggeration of six would call for them to stand at 3 upon 1; it would also call for scour holes having sides standing at physically impossible angles. The need for vertical exaggeration and the impossibility of making model sides have the erosive properties of the prototype's, force the model-maker into making formal vertical rigid sides if the model scale is small. Obviously very small scale models, where the depth becomes comparable with width are worthless; but even quite large ones are far from perfect, and quantitative showings near banks, or at rigid obstructions must be treated with distrust. The staff of a model should know about these troubles, and a host of others, so that they can apply theory tempered by practical judgment. They will expect to find that the model, when

manipulated to give most satisfactory results, will have scales slightly off the best calculations.

Scale Manipulations Calculated from Regime Theory

The following problem is devised from some figures and facts stated to relate to model running. Its object is to show how regime calculations might assist in planning the action that is necessary to move from the first approximation to model scales towards final satisfactory running conditions.

A tidal model has been constructed to the criteria (a) $F_b = 1.0$, which is essential for reproduction of tidal currents, (b) $F_s = 1.0$, which has some approximate theoretical justification,[8] and (c) a necessarily small width scale resulting in marked vertical exaggeration; the actual size of the model on the ground was large, so that adjustments of any kind called for considerable engineering effort. All other scales were calculated from regime theory. The sides were made vertical and rigid along the low-tide shore line taken from navigation charts. When the model was run first, with injection of sand where tidal action was not very great, bed movement near the ocean seemed reasonable, but waterlevels generally dropped more and more from expectation in the upstream direction, and injection sand deposited in a pile that moved forward very slowly. The model-maker, being aware of the weight of published opinion to the effect that model sand should be finer than the prototype's, decided that the trouble lay in the sand, so replaced it; when this produced no result it was replaced by even finer sand. As this did not achieve a cure added discharges were tried, till the surprising result was obtained that about 50% excess discharge gave bed-movement (initially) quite similar to that expected from the prototype's behavior.

The only obviously outstanding difference between the model as built and as "intended" lay in the vertical sides. Regime theory applies for waterlevel about low tide. Below low tide the so-called banks were of sand at comparatively flat slope. Study of sections suggested that the mean width below low-tide level, which seems more suitable for insertion in regime equations than either surface or bed-width, was reasonably assessible at between 5 and 10% tighter than surface width. Accepting 7.5% for calculation, one could say that the model as built, considered as a model of the model as designed, had imposed scales of $W = 1.075$, $F_b = 1.0$ if run properly, $T = 1.0$ unless the tidal and hydrographic control mechanisms were retimed, and $Q = 1.0$ unless the discharge control weirs were modified. As only three degrees of freedom actually exist, only three scales can be imposed; so the model maker could never make his model copy the prototype with the time scale and discharge scale fixed as they were.

The application of theory is simplified by Table 1, which arranges as column heads various useful trios of imposed scales, and as row titles scales that could be consequent; the calculations to make the table are exactly as in the preceding Sections but with symbols instead of numbers. The problem above may now be split into parts and solved quickly from the table thus:

Sub-problem 1.

Given $b = 1.075$, $F_b = 1.0$, $T = 1.0$. The scales given by the table will be those corresponding to the model as built being manipulated (by sand injection) with the original time-scale until agreement was obtained with prototype behavior. They are:

$$d = 1.075^2 = 1.15$$

$$S = 1/1.075^{0.5} = 0.965$$

and $$Q = 1.075^4 = 1.33$$

So, apparently the remarkably large excess of 33% on discharge would have to be run to compensate for 7.5% error in design of width. As a consequence the model slope would flatten only 3.5% and the ultimate depth would be 15% more than estimated originally. The amplitude of the tides being imposed on the model would have to be increased by 15% to make this change of depth scale effective.

Sub-problem 2.

Given b = 1.075, T = 1.0, Q = 1.0. The scales given by the table might give some indication of how the model would adjust to represent occurrences in the non-tidal upper reach near the point of injection, if what happened downstream did not affect the result. The only utility of such an artificial calculation is to try some method for showing why the sediment deposits did occur in the actual model; as the regime equations apply only to regime conditions they cannot tell directly what will happen during off-regime ones. The scales deduced are:

$$F_b = 1.075^4 = 1.33$$

$$d = 1/1.075^2 = 0.87$$

$$S = 1.075^{7/2} = 1.29$$

So, apparently, to get the upper end of the model to simulate the prototype requires injecting excessive sand so as to increase slope and decrease depth; and, of course, the sand injection would put the bed-factor up so that when the injected material reached the tidal reach the necessary conditions for reproduction there would be violated.

Sub-problem 3.

What could be achieved by replacing the coarse sand by finer? The answer to this seems to be "Nothing but unrepresentative rates of bed development." The dynamical requisite is $F_b = 1.0$. If this is achieved using prototype sand, then the charge will be the same as the prototype's; if it is achieved with finer sand then the charge will be greater than the prototype's.

Sub-problem 4.

b = 1.075, $F_b = 1.0$, Q = 1.0. The scales given by the table will indicate how to run the model with discharge as originally planned. They are:

$$T = 1.075^{4/3} = 1.10$$

$$d = 1./1.075^{2/3} = 0.95$$

$$S = 1/1.075^{1/6} = 0.99$$

This corresponds with what the author would call the "commonsense" solution of putting the model into initially scouring conditions, so that it could adjust quickly. But it is interesting to note that the time mechanism for the tides and river hydrograph should be run 10% slower.

Other problems may be solved on similar lines.

CONCLUSIONS

Reverting to the third paragraph of the Section, Models in General, the author suggests that:

1. An important model should be administered by a panel containing members who have experience of (a) rivers in general, (b) the river in question, (c) regime theory in practice, (d) models. One member may have several of these experiences.

2. The engineer in charge of running a mobile-bed river model should have had approved experience on river works and river observations plus approved knowledge of regime theory. Knowledge of rigid-boundary hydraulics alone should not be considered sufficient.

3. No important model should be undertaken without an assessment of how much of its projected work could be replaced by regime analysis of the river in question.

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Table I

Consequent Scales for Some Imposed Trios

(Kinematic Viscosity Scale assumed = 1.0)

	TQb	TbF_b	Tbd	QbF_b	bF_bS
F_b	$b^4/(QT^3)$	--	$b^2/(dT^2)$	--	--
F_s	b^2/T^3	b^2/T^3	b^2/T^3	QF_b/b^2	$F_b^{12}/(b^4S^{12})$
Q	--	$b^4/(F_bT^3)$	b^2d/T	--	$F_b^{11}/(b^2S^{12})$
b	--	--	--	--	--
d	QT/b^2	$b^2/(F_bT^2)$	--	$(Q^2/b^2F_b)^{1/3}$	$F_b^7/(b^2S^8)$
S	$b^{7/2}/(QT^{11/4})$	$F_bT^{1/4}/b^{1/2}$	$b^{2/3}/(dT^{7/4})$	$(F_b^{11}/b^2Q)^{1/12}$	--
T	--	--	--	$(b^4/F_bQ)^{1/3}$	b^2S^4/F_b^4
V	b/T	b/T	b/T	$(F_bQ/b)^{1/3}$	$F_b^4/(bS^4)$

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c. Discussion of several papers, grouped by Divisions.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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